

תשובות:

1. א. $y^2 = 36x$

ב. $\alpha = 18.434$

2. א. $t = \frac{1}{3}$

ב. $\overrightarrow{AK}$ תלוי ליניארית ב- $\overrightarrow{AG}$ ו- $\overrightarrow{AF}$ ויש לו נקודה משותפת לכן הוא נמצא על המישור שיוצרים $\overrightarrow{AG}$ ו- $\overrightarrow{AF}$.

3. א. $\ell_1 = \left(0, -24, \frac{-33}{2}\right) + t(1, 3, 2.5)$

ב. $S = 20\sqrt{65} = 161.24$

4. א. $\cos \theta + i \sin \theta$ (1)

$\cos \theta - i \sin \theta$

2θ (2)

ב. $S = \frac{-1}{16}\pi^2 + \frac{\pi}{4} + \ln \frac{\sqrt{2}}{2}$

5. א. $a = 1$

ב. $\frac{1}{2} < x < 4$

1.

$$y^2 = 2px \quad \text{א.}$$

דרך הנקודה (1,6)

$$36 = 2p \quad / : 2$$

$$p = 18$$

$$y^2 = 36x$$

$$r = \sqrt{(120-0)^2 + (-4-0)^2} = \sqrt{160}$$

$$x^2 + y^2 = 160 \quad y^2 = 36x$$

$$x^2 + 36x = 160$$

$$x^2 + 36x - 160 = 0$$

$$x_{1,2} = \frac{-36 \pm \sqrt{36^2 - 4(-160)}}{2} = \frac{-36 \pm 44}{2} \begin{matrix} 4 \\ -40 \end{matrix}$$

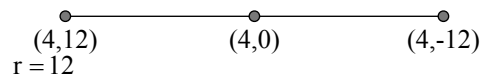
$$x = 4 \quad y^2 = 36 \cdot 4 = 144$$

$$y = \pm 12 \Rightarrow (4, 12), (4, -12)$$

$$x = -40 \quad y^2 = 36 \cdot (-40)$$

אין פתרון לשורש ריבועי שלילי

ב.



$$r = 12$$

$$(x-4)^2 + y^2 = 144$$

$$x^2 + y^2 = 160$$

$$y^2 = 160 - x^2$$

$$x^2 - 8x + 16 + 160 - x^2 = 144$$

$$-8x = -32$$

$$x = 4$$

$$y^2 = 160 - 16 = 144$$

$$(4, 12), (4, -12)$$

$$y^2 = 160 - x^2$$

$$y = \sqrt{160 - x^2}$$

$$y' = \frac{-2x}{2\sqrt{160 - x^2}} = \frac{-x}{\sqrt{160 - x^2}} = \frac{-4}{\sqrt{160 - 16}} = \frac{-4}{12} = -\frac{1}{3}$$

$$y^2 = 144 - x^2 + 8x - 16 = -x^2 + 8x + 128$$

$$y = \sqrt{-x^2 + 8x + 128}$$

$$y' = \frac{-2x}{2\sqrt{-x^2 + 8x + 128}}$$

$$y' = -2 \cdot 4 + 8 = 0$$

$$\tan \alpha = \frac{m_1 - m_2}{1 + m_1 \cdot m_2} = \frac{0 + \frac{1}{3}}{1} = \frac{1}{3}$$

$$\alpha = 18.434^\circ$$

2.

א.

$$\vec{AK} = \alpha \cdot \vec{AG} + \beta \cdot \vec{AF}$$

$$\vec{DG} = \frac{-\underline{v}}{2} + \frac{\underline{w}}{2} = \frac{\underline{w}}{2} - \frac{\underline{v}}{2}$$

$$\vec{AG} = \underline{v} + \frac{\underline{w}}{2} - \frac{\underline{v}}{2} = \frac{\underline{v}}{2} + \frac{\underline{w}}{2}$$

$$\vec{BF} = \frac{-\underline{u}}{2} + \frac{\underline{w}}{2} = \frac{\underline{w}}{2} - \frac{\underline{u}}{2}$$

$$\vec{AF} = \underline{u} + \frac{\underline{w}}{2} - \frac{\underline{u}}{2} = \frac{\underline{u}}{2} + \frac{\underline{w}}{2}$$

$$\vec{AK} = \alpha \cdot \left(\frac{\underline{v}}{2} + \frac{\underline{w}}{2} \right) + \beta \cdot \left(\frac{\underline{u}}{2} + \frac{\underline{w}}{2} \right) = \alpha \frac{\underline{v}}{2} + \alpha \frac{\underline{w}}{2} + \beta \frac{\underline{u}}{2} + \beta \frac{\underline{w}}{2} = \underline{v} \left(\frac{\alpha}{2} \right) + \underline{u} \left(\frac{\beta}{2} \right) + \underline{w} \left(\frac{\alpha}{2} + \frac{\beta}{2} \right)$$

$$\vec{EC} = -\underline{w} + \underline{u} + \underline{v}$$

$$\vec{EK} = -t\underline{w} + t\underline{u} + t\underline{v}$$

$$\vec{AK} = \underline{w} + t\underline{v} + t\underline{u} - t\underline{w} = w(1-t) + t\underline{v} + t\underline{u}$$

$$1-t = \frac{\alpha}{2} + \frac{\beta}{2} \quad t = \frac{\alpha}{2} \quad t = \frac{\beta}{2} \quad \alpha = \beta$$

$$t = \frac{\alpha}{2}$$

$$1 - \frac{\alpha}{2} = \frac{\alpha}{2} + \frac{\alpha}{2}$$

$$1 - \frac{1}{2}\alpha = 1$$

$$\alpha = \frac{2}{3}$$

$$t = \frac{1}{3}$$

ב.

$\vec{AK}$ תלוי ליניארית ב- $\vec{AG}$ ו- $\vec{AF}$ ויש לו נקודה משותפת ולכן הוא נצמא במישור שיוצרים $\vec{AG}$ ו- $\vec{AF}$.

3.

א.

מרחק (x, y, z) מהמישור π_1 הוא 3

$$\frac{2x + 4 - 2z}{\sqrt{2^2 + 1^2 + 2^2}} = 3$$

$$2x + y - 2z = 9$$

מרחק (x, y, z) מהמישור π_2 הוא 5

$$\frac{x - 2y + 2z}{\sqrt{1^2 + 2^2 + 2^2}} = 5$$

$$x - 2y + 2z = 15$$

$$+ \begin{cases} 2x + y - 2z = 9 \\ x - 2y + 2z = 15 \end{cases}$$

$$3x - y = 24$$

$$y = 3x - 24$$

$$x = t$$

$$y = 3t - 24$$

$$z = \frac{2x + y - 9}{2} = \frac{2t + 3t - 24 - 9}{2} = \frac{5t}{2} - \frac{33}{2}$$

$$\left(t, 3t - 24, \frac{5t}{2} - \frac{33}{2} \right) = \left(0, -24 - \frac{33}{2} \right) + t(1, 3, 25)$$

ב.

חיתוך xz הוא $y = 0$

$$3t - 24 = 0$$

$$t = 8$$

$$(8, 0, 3.5)$$

חיתוך yz הוא $x = 0$ $B(t = 0)$

$$B(0, -24, -16.5)$$

$$AB = \sqrt{(8 - 0)^2 + 24^2 + (3.5 + 16.5)^2} = 4\sqrt{65}$$

מרחק נקודה B מהמישור π_2

$$\left| \frac{0 - 2 \cdot (-24) + 2 \cdot (-16.5)}{\sqrt{1^2 + 2^2 + 2^2}} \right| = 5$$

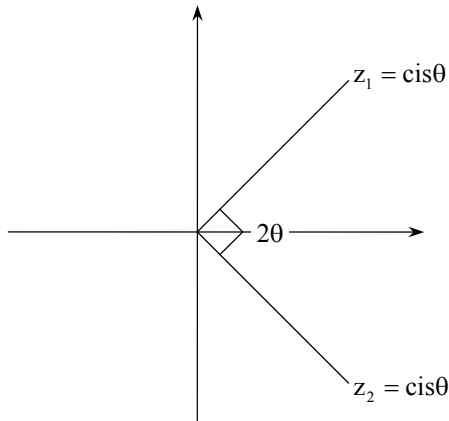
$$S = 5 \cdot 4\sqrt{65} = 20\sqrt{65} = 161.24$$

$$z^2 - (2\cos\theta)z + 1 = 0 \quad .4$$

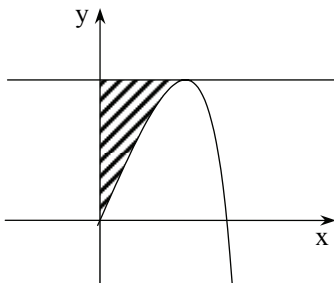
(1) .8

$$\begin{aligned} z_{1,2} &= \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \frac{2\cos\theta \pm \sqrt{4(\cos^2\theta - 1)}}{2} = \frac{2\cos\theta \pm \sqrt{-4(1 - \cos^2\theta)}}{2} = \\ &= \frac{2\cos\theta \pm \sqrt{-4\sin^2\theta}}{2} = \frac{2\cos\theta \pm \sqrt{4i\sin^2\theta}}{2} = \\ &= \frac{2\cos\theta \pm 2i\sin\theta}{2} \left\langle \begin{aligned} \frac{2\cos\theta + 2i\sin\theta}{2} &= \cos\theta + i\sin\theta \\ \frac{2\cos\theta - 2i\sin\theta}{2} &= \cos\theta - i\sin\theta \end{aligned} \right. \end{aligned}$$

(2)



.3



$$\begin{aligned} f(x) &= 2x - \frac{\sin x}{\cos x} \quad 0 \leq x < \frac{\pi}{2} \\ f\left(\frac{\pi}{2}\right) &= \frac{2\pi}{4} - \frac{\sin 45}{\cos 45} = \frac{\pi}{2} - 1 \end{aligned}$$

$$\int \frac{\sin x}{\cos x} dx \quad \cos x = t \quad ; \quad -\sin x dx = dt \quad \Rightarrow \quad dx = \frac{-dt}{\sin x}$$

$$\int \frac{\sin x}{t} \cdot \frac{-dt}{\sin x} = \int -\frac{1}{t} dt = -\ln t + c = -\ln(\cos x)$$

$$\int_0^{\pi/2} 2x - \frac{\sin x}{\cos x} dx + \int_{\pi/4}^0 \frac{\pi}{2} - \ln t dx = \frac{2x^2}{2} + \ln(\cos x) \Big|_0^{\pi/4} + \frac{\pi}{2} x - x \Big|_{\pi/4}^0 =$$

$$\left(\frac{\pi^2}{16} + \ln \frac{\sqrt{2}}{2} \right) - (0) + (0) - \left(\frac{\pi}{2} \cdot \frac{\pi}{4} - \frac{\pi}{4} \right) = \frac{\pi^2}{16} + \ln \frac{\sqrt{2}}{2} - \frac{\pi^2}{8} + \frac{\pi}{4} = -\frac{1}{16} \pi^2 + \frac{\pi}{4} + \ln \frac{\sqrt{2}}{2}$$

.5

א.

$$f(x) = \log_{\frac{1}{2}} x + a \log_{\frac{1}{2}} (6-x)$$

$$f'(x) = \frac{1}{x \cdot \ln \frac{1}{2}} + \frac{a(-1)}{(6-x) \cdot \ln \frac{1}{2}} =$$

$$f'(4) = \frac{1}{4 \cdot \ln \frac{1}{2}} - \frac{a}{2 \cdot \ln \frac{1}{2}} = \frac{1-2a}{4 \cdot \ln 2^{-1}} = \frac{1-2a}{-4 \ln 2}$$

$$f(4) = \log_{\frac{1}{2}} 4 + a \log_{\frac{1}{2}} 2 = -2 + a(-1) = -2 - a$$

$$(4, -2-a), m = \frac{1-2a}{-4 \ln 2}$$

$$y - (-2-a) = \frac{1-2a}{-4 \ln 2} (x-4)$$

נציב $y = 0$

$$0 + 2 + a = \frac{1-2a}{-4 \ln 2} x + \frac{4(1-2a)}{4 \ln 2}$$

$$2 + a = \frac{2a-1}{4 \ln 2} x + \frac{1-2a}{\ln 2} \quad / \cdot 4 \ln 2$$

$$8 \ln 2 + 4a \ln 2 = (2a-1)x + 4(1-2a)$$

$$8 \ln 2 + 4a \ln 2 = (2a-1)x + 4 - 8a$$

$$8 \ln 2 + 4a \ln 2 + 8a - 4 = (2a-1)x$$

$$8 \ln 2 + 4a \ln 2 + 8a - 4 = (2a-1)(4 + 12 \ln 2)$$

$$8 \ln 2 + 4a \ln 2 + 8a - 4 = 8a - 4 + 24a \ln 2 - 4 - 12 \ln 2$$

$$20 \ln 2 = 20a \ln 2 \quad / : 20 \ln 2$$

$$a = 1$$

ב. תחום הגדרה: $x > 0$

$$x^{\left(\log_2 x + \log_{\frac{1}{4}} 4 \right)} < 4 = \log_4 x^{\left(\log_2 x + \log_{\frac{1}{4}} 4 \right)} < \log_4 4 = \left(\log_2 x + \log_{\frac{1}{4}} 4 \right) \cdot \log_4 x < 1 =$$

$$(\log_2 x - 1) \frac{1}{\log_4 x} < 1 = \frac{\log_2 x - 1}{2 \log_x 2} < 1 = \frac{(\log_2 x - 1) \log_2 x}{2} < 1$$

$$t = \log_2 x$$

$$\frac{t(t-1)}{2} < 1$$

$$t^2 - t - 2 < 0$$

$$t^2 - t - 2 = 0$$

$$t_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{matrix} 2 \\ -1 \end{matrix} \Rightarrow -1 \leq t < 2$$

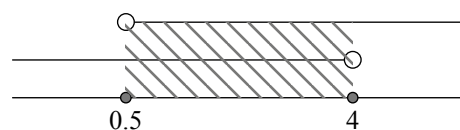
$$-1 < \log_2 x < 2$$

$$\log_2 x < 2$$

$$\log_2 x > -1$$

$$x < 4$$

$$x > 2^{-1} \rightarrow x > \frac{1}{2}$$



$$\frac{1}{2} < x < 4$$